**UNIT-I**

**REVIEW OF DISCRETE SIGNALS AND SYSTEMS**

**Signal: A Signal is defined as any physical quantity that varies with time, space or any other independent variables.**

**Eg: (t)=2t linear with independent of t.**

**ECG(Electro Cardiogram) signal can carry information.**

 **A signal can be function of one or more independent variables. If the signal function depends only one variable, then it is called ”one dimensional Signal”.**

 **AC power supply signal and speed signal are functions of one independent variable namely time.**

 **If the signal depends on two independent variables, then the signal is known as “Two dimensional Signal”.**

 **A Picture is a function of two independent variables.**

**Classification of Signals:**

**There are three types of signals, namely**

**1.) Analog Signals**

**2.) Discrete Signals**

**3.) Digital Signals**

**Analog (or) Continuous time signal:**

**A Signal that varies continuously with time is known as “Continuous time signal”.**

**Eg: Speed Signal and temperature of room.**

**Discrete time Signal:**

**The signal that are defined at discrete instants of time are known as “Discrete time signal”.**

**Eg: Train traffic signal.**

**Digital Signal:**

**Digital signals are signals whose both dependent variable and independent variable are discrete in nature. Digital signals comprise of pulses occurring at discrete intervals of time.**

**Eg : Telegraph and Teleprinter signals.**

**Signal Processing:**

**Processing: To obtain the signal in the more desirable form.**

 **By changing the basic nature of signal to obtain the desired shaping of the input signal is called “Signal processing”.**

 **Signal processing is of two types depending on the type of signal to be processed.**

1. **Analog signal processing**
2. **Digital signal processing**

**1.) Analog Signal Processing:**

 **In Analog signal processing, Continuous amplitude and Continuous time signals are processed.**

 **Various types of analog signals are processed through low pass filters, high pass filters, band pass and band reject filters. To obtain the desired shaping of input signal.**

**System:**

 **A System is defined as a physical device that generates are -----or an output signal for a given input signal.**

**Signal Processing Systems:**

 **Signal processing systems are of two types depending upon the type of signal to be processed.**

**\*Continous time system:**

 **Continous time systems are the system for which both input and output are continous time signals.**

**\* Discrete time system:**

 **Discrete time system are the system for which both input and output are discrete time signals.**

**Representation of discrete time signals:**

 **There are different types of representations for discrete time signals. they are,**

 **1.Graphical representation**

 **2.Functional representation**

 **3. Tabular representation**

 **4. sequence representation**

 **1. Graphical representation:**

**Let us consider a signal x(n) with values x(-1)=1,x(0)=2,x(1)=1,x(2)=0.5,x(3)=1.5. The discrete time signal can be represented graphically as shown ,**

**2. Functional Representation:**

 **The digital signal can be represented functionally as,**

**3. Tabular Representation:**

 **The digital signal can be represented in tabular for as shown below,**

**4. Sequence Representation:**

 **A finite duration sequence with tie origin(n=0) indicated by symbol ↑ can be represented as,**

**X(n)={1,2,2,0.5,1.5}**

 **↑**

**Elementary Discrete Time Signals:**

1. **Unit Step Response:**

**The unit step response is defined as, u (n) = {1 n≥0**

 **0 n<0**

 **The graphical representation for the unit step response is,**

1. **Unit Ramp Response:**

**The unit ramp response is defined as, r (n) = {n n≥0**

 **0 n<0**

 **The graphical response for the unit ramp response is shown below,**

1. **Unit Impulse Response:**

**The unit impulse response is defined as, s (n) = {1 n=0**

 **0 n≠0**

 **The graphical response for the unit impulse response is as shown,**

1. **Exponential sequence:**

**The exponential signal is the sequence of the form, x (n) = for all n.**

 **The below figure shows different types of discrete time exponential signals.**

 **When the value is 0<a<1, the sequence decays exponentially and for a<-1 and -1<a<0 the discrete time exponential signals as shown below.**

**Operation on Signals:**

 **The basic set of operations depends on,**

* **Shifting**
* **Time reversal**
* **Time scaling**
* **Scalar multiplication**
* **Signal multiplier**
* **Signal addition**
1. **Shifting:**

**The shifting operation takes the input sequence and shifts the value by an integer increment of independent variable.**

 **Mathematically this can be represented as,**

 **Y (n) = x (n-k).**

**Where x (n) is input and y (n) is the output. If k is +ve the shifting delays the sequence and if k is –ve the shifting advances the sequences.**

 **A signal x (n) is shown in below fig, the signal x (n-3) is obtained by shifting**

**x (n) right by 3 units of time. The result is as shown in fig (a). On the other hand, the signal x(n+2) is obtain by shifting x(n) left by 2 units of time. The result is as shown in below fig.**

1. **Time reversal:**

**Let us consider x (n) is an input sequence. The output sequence y (n) for time reversal is, y (n) = x (n-1).**

**For the signal x (n), is shown in fig (2), the signal x (+n) is given in fig 2 (a).**

1. **Time scaling:**

**This is accomplished by replacing ‘n’ by ‘xn’ in sequence x (n). Let x (n) is a sequence as shown in fig 3. If λ=2 we get a new sequence y (n) = x (2n).**

 **We can plot the sequence y (n) by substituting different values for “n”.**

 **Y (n) = x (2n).**

**n=0, y (0) = x (0) =3**

**n=1, y (1) = x (2) =3**

**n=2, y (2) = x (4) =1**

**n=-1, y (-1) = x (-2) =3**

**n=-2, y (-2) = x (-4) =1.**

**The o/p sequence, y (n) = x (2n) is as shown in fig 3a.**

**Skip the odd numbers in the o/p sequence compared to i/p sequence.**

1. **Scalar multiplication:**

**A scalar multiplication is shown in fig 4. Hence the signal x (n) multiplied by a scale factor “a”.**

**Eg: if x (n)={1,2,3,4} and a=3 then the signal y (n) = a X(n) = 3 X(n) ={3,6,9,12}.**

1. **Signal multiplier:**

**Fig 5 illustrates the multiplication of two signals to form another sequence.**

**Eg: if x1(n)={1,2,3,4}, x2(n)={1,-1,1,-1} then the signal y(n) = x1(n)\*x2(n) = {1,-2,3,-4}.**

1. **Signal addition:**

**Two signals can be added by using an adder as shown in fig 6.**

**Eg: if x1(n) = {1,2,3,4} , x2(n)= {1,-1,1,-1} then the signal y(n) = x1(n) + x2(n) = {2,1,4,3}.**

**Classification of Discrete Time Systems:**

 **According to properties and characteristics , the discrete time systems can be classified into 6 systems.**

* **Static and dynamic systems**
* **Casual and non-casual systems**
* **Linear and non-linear systems**
* **Time varient and invarient systems**
* **FIR and IIR systems**
* **Stable and unstable systems**

**Casual and non casual system:**

**A system is said to be casual if the o/p of the system at any time ‘n’ depends only at present and past inputs but doesn’t depends on future inputs. These can be represented mathematically as, y(n) = { x(n1), x(n-1),x(n-2),--------}**

 **If the o/p of the system depends on the future i/p the system is said to be “non casual systems”.**

**Eg: y (n) = x (n) + x (n-1)🡪 casual system**

 **Y (n) = x (2n)🡪non casual system.**

**Pb. Determine if the system described by the following equations are casual/non casual**

**1.Y (n) = x (n) + 1/x (n-1)**

 **Sol: Given y (n) =x (n) +**

 **Let n=0, y (0)= x(0)+**

 **n=1, y (1)= x(1)+**

 **n=2 , y(-1)= x(-1)+**

**for different values of ‘n’ the o/p system only depends on present and past input systems, then the given system is “casual system”.**

**2.Y(n) = x(**

**Sol: Given y(n) = x()**

 **Let n=0, y(0) = 0**

 **n=1, y(1) = x(1)**

 **n=2, y(2) = x(4)**

 **n=-1, y(-1) = x(1)**

 **n=-2, y(-2) = x(4)**

**for different values of ‘n’ the output of n only depends on future inputs. So it is non casual system.**

**Linear and Non-Linear Systems:**

**If the System satisfies the superposition principle , then the system is called Linear system .**

**A System is linear if and only if T[a1x1(n)]+T[a2X2(n)]=T[a1X1(n)+a2X2(n)]**

**Where a1 and a2 are arbitrary constants**

**A System doesn’t satisfy the superposition principle is called “non-linear system”.**

**1.determine if the system described by following i/p –o/p equations is linear or non-linear, y(n) = x(n)+**

**Let (n) are two input sequences then,**

**(n) = + 🡪①**

**(n) = + 🡪②**

**(n)] = + 🡪③**

**(n)] = + 🡪④**

 **(n)] = T [ ]**

 **= + 🡪⑤**

**③+④ + + + + 🡪⑥**

**Eq ⑤ and ⑥are not equal then above eqn is not satisfy the super position principle then system is non linear.**

**Time-variant and Time invariant Systems:**

**A System is said to be time invariant or shift invariant, if the characteristics of the system doesn’t change with time.**

 **For the time invariant system, if y(n) is the response of the system to the input x(n) then the response of the system to the input x(n-k) is y(n-k).**

**Representation of an arbitrary system:**

**Any arbitrary sequence x(n) can be represented as shown in fiq1. The sample x(0) can be obtained by multiplying x(0) with unit impulse as shown in fiq2**

**The sample x(1) with δ(n-1) as shown in fiq3**

**The sample x(2) can be obtained by multiplying x(2) with δ(n-2) as shown in fiq4**

**The sample x(-1) can be obtained by multiplying x(2-1) with δ(n+1) as shown in fiq5**

**The sample x(-2) can be obtained by multiplying x(-2) with δ(n+2) as shown in fiq6**

**The sum of five sequences , we get x(n) as shown in fiq1**

**Similarly, for getting ‘k’ sequence the value of x(n) is**

 **X(n) =**

**Z-Transform:**

**The z-transform of a discrete time signal x(n) is defined as**

 **Z[x(n)]=X(Z)=x(n). →→①**

**Where “Z” is a complex variable.**

**In polar form, the Z can be represented as Z=r. →②**

**Where r=radius of the circle**

 **W=angle of the circle**

**Sub , eq② in eq①, we get**

**X(Z)=**

**Region of Convergence (ROC):
The portion of the Z-plane for which the series in equ.1 converges is called “Region of Convergence”. The ROC depends on the magnitude of Z. The ROC can be a circle , interior of a circle, exterior of a circle, an annulus and the entire Z-Plane.**

**Possible configurations of ROC for Z-Transform:**

**These configurations are**

 **Interior of Circle**

 **Exterior of Circle**

 **Annulus of Circle**

 **Entire Z-plane**

**Z-Transform and ROC of Finite duration Sequence:**

**Right hand sequence:**

 **A signal that has finite duration on right hand sides is known as right hand sequence. For such type of sequence, the ROC is entire Z-plane except Z=0.**

1. **Find Z-transform and Roc of casual sequence x(n) = {1,0,3,-1,2}**

**Given, x(n) = {1,0,3,-1,2}**

**x(Z)=**

 **= + + + +**

 **= 1 + 0 +**

 **= 1 +**

**The x(Z) converges for all values of Z except at Z=0.**

**Left hand sequence:**

 **A signal that has finite duration on left hand sides is known as right hand sequence. For such type of sequence, the ROC is entire Z-plane except Z=.**

1. **Find the Z-transform and ROC of the anti-casual sequence x(n)= {-3,-2,-1,0,1}**

**Given x(n)= {-3,-2-1,0,1}**

**X(Z)=**

**=**

**=**

**The X(Z) converges for all values of Z except at Z=.**

**Two sided sequence:**

 **A signal that has finite duration on both left and right hand sides is known as “Two sided sequence”. For such type of sequence, the ROC is entire Z-plane except Z=0 and Z=.**

1. **Find the Z-transform and ROC of sequence x(n)= {2,-1,3,2,1,0,2,3,-1}**

 **Given x(n) = {2,-1,3,2,1,0,2,3,-1}**

 **x(Z)=**

 **=**

 **=**

**The ROC of above sequence is entire Z-plane except Z=0 and.**

**Z-Transform and ROC of Infinite Duration Sequence:**

1. **Determine the Z-transform and ROC of the signal x(n)=U(n)**

 **The given signal is casual and infinite duration signal.**

**The Z-transform of x(n) is given by,
 X(Z) =**

 **=**

 **=**

 **=**

 **X(Z) =**

**This is a geometric series for infinite no. of points**

 **=**

 **Converges for |a|<1**

**>a**

 **of sequence x(n)is the exterior of the circle having radius ‘a’ as shown in fig.**

**Two Sided Sequence:**

**From the above equation we can find that the ROC of a casual signal is exterior of a circle of radius ‘a’ and ROC of anti casual signal is interior of circle of radius ‘b’.**

 **Now let us consider a two sided sequence is,**

 **X(n)= U(-n-1)**

**The Z-transform of x(n) is,**

 **X(Z) =**

 **= X(Z) =**

 **=**

 **=**

1. **The first power series converges is |a|<1**

**|Z|>a**

1. **The second power series converges is |Z|<1**

**|Z|<b**

 **If b is <a then |b|<|a| the two ROC’s don’t overlap as shown in fig1. x(Z) doesn’t exist . if |b|>|a| the two ROC’s don’t overlap as shown in fig2. x(Z) exist . so ROC of X(Z) is |a|<|Z|<|b| i.e., for an infinite duration two sided signal the ROC is a ring in Z-plane.**

 **X(Z) =**

 **ROC is |a|<|Z|<|b|.**

**Properties of Z-Transform:**

**1.Linearity Property:**

**If X1(Z)=Z[x1(n)] and X2(Z)=Z[x2(n)] then Z[ax1(n)+bx2(n)]=aX1(Z)+bX2(Z).**

**Proof: Z[a(n)] =**

 **=**

**: Z[a(n)] = aX1(Z)+bX2(Z).**

**2.Time shifting property:**

**If X(Z)=Z[x(n)] then Z[x(n-m)]=Z-mX(Z).**

**Proof: Z[x(n)] =**

 **Z[x(n-m)] =**

 **=**

 **=**

**Let**

 **=**

 **=**

**3.Multiplication Property:**

**If X(Z)=Z[x(n)] then Z[anx(n)]=X(a-1Z).**

**Proof: Z[x(n)] =**

 **Z[x(n)] =**

 **=**

 **= X()**

 **Z[x(n)] = X()**

**4.Time reversal property:**

**If X(Z)=Z[x(n)] then Z[x(-n)]=X(Z-1).**

**Proof: Z[x(n)] =**

 **Z[x(-n)] =**

**Let –n=l**

 **Z[x(-l)] =**

 **=**

**Z[x(-n)] = x(**

**5.Differentiation property:**

**If X(Z)=Z[x(n)] then Z[n.x(n)]=-Z.**

**Proof: Z[x(n)] =**

**Taking differentiation on b.s., w.r.t Z,**

 **X(Z) =**

 **= -**

**Multiplying ‘-Z’ on b.s, we get**

**-Z X(Z) =**

 **-Z X(Z) = Z[nX(n)]**

**Properties of ROC:**

**\*the ROC cannot contain any poles.**

**\* If x(n) is a casual sequence then the ROC is the entire Z-plane except at Z=0.**

**\* if x(n) is non-casual sequence then the ROC is the entire Z-plane except at Z=.**

**\* if x(n) is finite duration, two sided sequence the ROC is entire Z-plane except at Z=0 and Z=**

**\* if x(n) is an finite duration, two sided sequence the ROC will consists of a ring in the Z-plane, bounded on the interior and exterior by a pole.**

**\* the ROC of a linear time invariant stable system contains the unit circle.**

**\* the ROC must be connected region.**

**System Function:**

 **In general the system is, described by a linear constant coefficient differential equation of the form,**

 **Y(n)=- y(n-1) + x(n-1) →①**

**Taking Z-transform on B.S and applying time shifting property, we obtain**

 **Y(Z)=- y(Z). + x(Z).**

 **Y(Z)+ y(Z). = x(Z).**

 **Y(Z) [1+ = x(Z).**

 **H(Z) = = →②**

**1. Find the system function and impulse response of the system described by the differential equation y(n )= y(n-1) + x(n)**

**Sol: given y(n )= y(n-1) + x(n).**

**Taking Z-transform and apply time shifting property**

 **Y(Z)= y(Z) .+ x(Z)**

 **Y(Z) - y(Z) . = x(Z)**

 **Y(Z)[1 - .] =x(Z)**

 **H(Z) = =**

 **Applying inverse Z-transforms,**

 **h(Z)=**

**h(n)= u(n)**

**2. y(n)= x(n) + 2x(n-1) – 4x(n-2) + x(n-3)**

 **Given y(n)= x(n) + 2x(n-1) – 4x(n-2) + x(n-3)**

**Taking Z-transform and apply time shifting property**

**y(Z)= x(Z) + 2x(Z) – 4x(Z). + x(Z)**

 **= X(Z) [1+2-4+]**

**H(Z)= = 1+2-4+ ====🡺 {1,2,-4,1}**

 **=h(0) + h(1) + h(2). + h(3).**

**Where h(0)=1, h(1)=2, h(2)=-4, h(3)=1**

**3. y(n)= y(n-1) +2y(n-2) + x(n)**

**Given, y(n)= y(n-1) +2y(n-2) + x(n)**

**Taking Z-transform and apply time shifting property**

**y(Z)= y(Z). +2y(Z). + x(Z)**

 **y(Z) [1--2] = x(Z)**

 **h(Z) =**

**Applying Z-inverse transform,**

 **=**

 **=**

 **= .**

1. **Y(n)= 2x(n-1) +1/2 x(n-2) + x(n-3) + 1/3 x (n-4)**

 **Given, Y(n)= 2x(n-1) +1/2 x(n-2) + x(n-3) + 1/3 x (n-4)**

 **Taking Z-transform and apply time shifting property**

 **Y(Z)= 2x(Z). +1/2 x(Z) .+ x(Z) .+ 1/3 x (Z).**

 **H(Z)= 2 + 1/2++ 1/3**

**Convolution Property:**

 **If X(Z)= Z[x(n)] and H(Z)= Z[h(n)] then Z[x(n).h(n)]=X(Z).H(Z)**

**Where x(n).h(n) denotes the linear convolution of sequences.**

**Proof:**

 **y(n)=**

**Apply Z-transform from above equation, we get**

**Z[y(Z)] =**

 **=**

**Let n-k=l**

 **=**

 **=x(Z).H(Z)**

**Initial Value Theorem:**

 **If x+(Z)= Z[x(n)] then X(0)=**

**Proof:**

 **Z[x(n)] =**

 **X+(Z)=**

**As Z→∞ all the terms vanished except x(0) which proves the theorem**

 **Apply limits on B.S, then**

 **=**

 **=**

 **=**

 **=x(0)**

 **X(0)=**

**Final Value Theorem:**

 **If ) =Z[x(n)], where the ROC for x(Z) includes, then x(∞)= .**

**Proof:**

 **Z[x(n+1)]-Z[x(n)] =**

 **Z x+(Z)-Zx(0)-x+(Z) =**

 **(Z-1) x+(Z)-Zx(0) =**

 **=**

 **(Z-1) x+(Z)-Zx(0) = x(1)-x(0)+x(2)-x(1)+x(3)-x(2)+.......+x(∞+1)-x(∞)**

 **(Z-1) x+(Z)-Zx(0) = -x(0)+x(∞)**

**Let Z→1, we get**

 **= -x(0)+x(∞)**

 **= x(0)+x(∞)**

 **X(∞)=**

**Problems:**

**1.Find Z-transform of a signal x(n) = [3(-4(]u(n)**

 **Given x(n) = [3(-4(]u(n)**

 **Z[x(n)] =**

 **Z [3(-4(]u(n) =**

 **=**

 **=3. -4**

 **=3.-4**

 **=3]-4[]**

 **=**

**2. Find the Z-transform of the signal x(n)= u(n)**

 **Given x(n)= u(n)**

 **Z[(n)] = x(Z) = .**

 **Z[ u(n)] = .**

 **=.**

 **=**

 **=1/2 [. + .]**

 **=1/2 [+ ]**

 **=1/2 []**

 **=1/2[]**

 **= =**

**3. find Z-transform of the signal x(n) = . u(n)**

 **Given x(n) = . u(n); 0<<1**

 **X(Z) = Z[x(n)] =**

 **=**

 **=**

 **= [] (**

 **= [( - (]**

 **= [( - (]**

 **= [. - .]**

 **= [. - .]**

 **= [ - ]**

 **= []**

 **= []**

 **X(Z) =**

**4. x(n) = u(n-1)**

**Given x(n) = u(n)**

 **X(Z) =**

 **=**

 **=**

 **=**

 **Z[u(n-1)] = .**

 **=**

**5. x(n) = U(n)**

 **Z [ U(n)] =**

**By using multiplication of exp.sequence property, then**

**Z [ U(n)] =**

 **=**

**6. x(n) = n. U(n)**

**Z [ U(n)] =**

 **= =**

 **=**

**By using differentiation property,**

**Z[n.x(n)] = -Z.x(Z)**

**Z[n.U(n)] = -Z. []**

 **= -Z- a**

 **= =>**

**Z-Transforms and ROC [Problems]:-**

**(a).Find the Z-transform of the following discrete time signals and find ROC for each.**

1. **X(n)= U(n)+5U(-n-1)**

**Z[x(n)] =**

 **= +**

 **= + 5**

 **= +5()**

 **= +**

 **ROC :: ||<1 and |2|<1**

 **|Z|> and |Z|< 2**

 **<|Z|<2**

**(b). x(n) =**

 **Z[x(n)] = + -**

**The ROC for above sequence x(n) is entire Z-plane except at Z=0.**

**(c). x(n) = U(n-2)**

 **Z[U(n)] =**

 **Z[U(n-2)]=**

 **=**

 **=**

**ROC of the above sequence is |Z|>1**

**(d). x(n) = (n+0.5) U(n)**

**Z[(n)] =**

 **= +**

 **=**

 **=**

**By using differentiation property,**

 **U(n) = -Z ()**

 **=-Z. \***

 **=**

**= = 0.5**

 **= +0.5.**

**|| < 1 and |Z|>**

**(e).X(n) = U(n)**

**Z[x(n)] =**

 **=**

 **= [ - ]**

 **= []**

 **= []**

 **=**

 **= []**

**The ROC is >1 and <1**

**(f). x(n) =**

 **Z[U(n)] =**

 **=**

 **X(z)= =>**

**=[n. = -Z []**

 **=-Z .a.**

 **=**

 **=**

**Z[ = -Z x(Z)**

 **= -Z**

**Inverse Z-Transform:**

 **There are four methods that are used for the evaluation of inverse Z-transform.**

1. **Long division method**
2. **Partial fraction method**
3. **Residue method**
4. **Convolution method**

**1.Long division method:-**

**For a rational Z-transform, a power series expansion can be obtained by long division method. For this method we have a Z-transform x(z) with its corresponding ROC.**

 **Now, we can expand x(z) into a power series of the form,**

 **X(Z)=**

**Which converges the given ROC**

**Inverse Z-transform of X(Z) is given by**

 **X(n)= =**

**1.Find the Inverse Z-transform of X(Z)= IZI>1 by using long division method ?**

 **Sol:**

 **Given X(Z)= IZI>1**

**From the ROC, we get that x(n) is casual sequence**

 **Z+0.2**

 **Z-0.5-0.5**

 **0.7+0.5**

 **0.7-0.35-**

 **0.85+0.35**

 **0.85**

 **0.775**

 **0.775**

 **0.812**

**X(z)=+……………………….**

**Apply inverse Z-transform , then**

**x(n)={x(0),x(1),x(2),x(3),…………………………..}**

**x(n)={ 0 , 1 , 0.7 , 0.85 , 0.775 ,……………..…..}**

**2.Find the inverse Z-transform 0f X(Z)= ; IZI<3 by using long division method ?**

**Sol:**

 **Given X(Z)= ; IZI<3**

**From the ROC, we get that x(n) is anti-casual sequence**

**12-7Z+ Z**

 **Z-+**

 **+**

**X(Z) = …………………………**

**Apply inverse Z-transform , then**

**x(n)={…………………………..,x(-2),x(-1),x(0)}**

**x(n)={ ……………..….. , , , , 0 }**

**2.Partial fraction expansion method:**

 **In this method, the function X(Z) is the linear combination of ,,……………..**

**Where ,,………….are additive parts of function X(Z) recovered by partial function it is as shown in equ.1**

**X(Z)=,,,……………………………., ①**

**Where ,,…………. are the expressions with inverse Z-Transform.**

 **then inverse Z-transform of X(Z) will be**

 **x(n)=**

 **= ,,,…………………………….,]**

 **=]+]+…………….+]**

 **x(n)= ,,,……………………………., ②**

**This method is particularly useful if X(Z) is a rational transfer function given as follows**

**X(Z)== ③**

**Here should be equal to 1 []**

**The partial fraction method is applicable only for proper rational transfer function.**

**A rational transfer function is called Proper if and M<N.**

**1.Determine the inverse –transform by using partial fraction expansion method.Given**

 **?**

**Sol:**

 **Given ①**

 **=**

 **If Z = , then A = 2**

**If Z = , then B = -1**

 **=**

 **= 4 [**

**3.Residue Method:**

 **The Z-Transform of a sequence x(n) is given by**

 **X(Z) = ①**

**Multiplying on both sides and integrating w.r.t Z about a closed contour C in the ROC of X(Z) then**

 **=**

 **=**

**By using Cauchy residue theorem,**

**Where**

 **= {0 n ≠ k**

**Then**

 **= for n=k**

**The inverse Z-Transform relation is given by**

 **x(n) =**

**where C is a circle in Z-plane in ROC of X(Z).**

**the equ.6 can be evaluated by finding sum of all residues of poles that are inside a circle C .Now, equ.6 can be written as**

**X(Z) has no poles inside the circle C for one of more values of n then x(n) is 0.**

**1.Using residue method, find the inverse Z-Transform of IZI>1 ?**

**Sol:**

 **Given**

**For IZI >1 then**

**Convolution Method:**

 **In this method the given X(Z) is split into such that X(Z)=**

**Next we find by taking inverse Z-transform of respectively.**

**From convolution theorem property of Z-transform**

 **Z [] = = X(Z) →**

**From eq①, we know that convolution of,is the inverse Z-transform of X(Z).**

**2.Find the inverse Z-transform of X(Z) = by using convolution theorem**

**Sol: Given X(Z) =**

 **= =**

**Here, = and =**

 **X(n)= =U(n)\***

 **=**

 **=**

 **=**

 **= []U(n)**

 **= [-2-]U(n)**

 **=[2-]U(n)**

 **= []U(n)**

 **= []U(n)**